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| **HS Mathematics: Statistics & Probability Overview – Planning Tool**  |
| Collaborators:  | Academic Year: |
| *This planning tool can be used by collaborating teachers across a given school year or term to help insure full implementation of the Iowa Core Content Standards into their classroom instructional and assessment activities.* *Full implementation is accomplished when the district or school is able to provide evidence that an ongoing process is in place to ensure that each and every student is learning the standards and the essential concepts and skills of the Iowa Core. A school that has fully implemented the Iowa Core is engaged in an ongoing process of data gathering and analysis, decision making, identifying actions, and assessing the impact around alignment and professional development focused on content, instruction, and assessment. The school is fully engaged in a continuous improvement process that specifically targets improved student learning and performance.* ***Effective implementation of the Iowa Core is not a simple checklist. Implementation requires that educators strategically and systematically address the knowledge and skills being taught, engage in collaboration around the use of effective instructional practices and materials and develop activities to elicit evidence of student learning that match the level of rigor called for in the standards.*** |
| **Mathematic Content Standard** | **Aug.** | **Sept** | **Oct.** | **Nov.** | **Dec.** | **Jan.** | **Feb.** | **Mar** | **Apr.** | **May** |
| **Interpreting Categorical and Quantitative Data: Summarize, represent, and interpret data on a single count or measurement variable** |
| 1. Represent data with plots on the real number line (dot plots, histograms, and box plots). **(S-ID.1.) (DOK 1,2)**
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| 1. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. **(S-ID.2.) (DOK 1,2)**
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| 1. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). **(S-ID.3.) (DOK 1,2)**
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| 1. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. **(S-ID.4.) (DOK 1,2)**
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| **Interpreting Categorical and Quantitative Data: Summarize, represent, and interpret data on two categorical and quantitative variables**  |
| 1. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. **(S-ID.5.) (DOK 1,2)**
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|  | **Aug.** | **Sept** | **Oct.** | **Nov.** | **Dec.** | **Jan.** | **Feb.** | **Mar** | **Apr.** | **May** |
| 1. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. **(S-ID.6.) (DOK 1,2)**
2. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.
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| 1. Informally assess the fit of a function by plotting and analyzing residuals.
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| 1. Fit a linear function for a scatter plot that suggests a linear association.
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| **Interpreting Categorical and Quantitative Data: Interpret linear models** |
| 1. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. **(S-ID.7.) (DOK 1,2)**
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| 1. Compute (using technology) and interpret the correlation coefficient of a linear fit.  **(S-ID.8.) (DOK 1,2)**
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| 1. Distinguish between correlation and causation. **(S-ID.9.) (DOK 1,2)**
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| **Making Inferences and Justifying Conclusions: Understand and evaluate random processes underlying statistical experiments** |
| 1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population. **(S-IC.1.) (DOK 1)**
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| 1. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model? **(S-IC.2.) (DOK 1,2)**
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| **Making Inferences and Justifying Conclusions: Make inferences and justify conclusions from sample surveys, experiments, and observational studies**  |
| 1. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. **(S-IC.3.) (DOK 1,2)**
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| 1. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. **(S-IC.4.) (DOK 2)**
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|  | **Aug.** | **Sept** | **Oct.** | **Nov.** | **Dec.** | **Jan.** | **Feb.** | **Mar** | **Apr.** | **May** |
| 1. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. **(S-IC.5.) (DOK 2,3)**
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| 1. Evaluate reports based on data. **(S-IC.6.) (DOK 2,3)**
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| **Conditional Probability and Rules of Probability: Understand independence and conditional probability and use them to interpret data** |
| 1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not") **(S-CP.1.) (DOK 1,2)**
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| 1. Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent. **(S-CP.2.) (DOK 1)**
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| 1. Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B**. (S-CP.3.) (DOK 1,2)**
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| 1. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results. **(S-CP.4.) (DOK 1,2)**
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| 1. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer. **(S-CP.5.) (DOK 1,2,3)**
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| **Conditional Probability and Rules of Probability: Use the rules of probability to compute probabilities of compound events in a uniform probability model** |
| 1. Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model. **(S-CP.6.) (DOK 1,2)**
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| 1. Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model. **(S-CP.7.) (DOK 1,2)**
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| 1. (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model**. (S-CP.8.) (DOK 1,2)**
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| 1. (+) Use permutations and combinations to compute probabilities of compound events and solve problems. **(S-CP.9.) (DOK 1,2)**
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| **Using Probability to Make Decisions: Calculate expected values and use them to solve problems** |
| 1. (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. **(S-MD.1.) (DOK 1,2)**
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| 1. (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. **(S-MD.2.) (DOK 1,2)**
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| 1. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes. **(S-MD.3.) (DOK 1,2,3)**
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| 1. (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households? **(S-MD.4.) (DOK 1,2,3)**
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| **Using Probability to Make Decisions: Use probability to evaluate outcomes of decisions** |
| 1. (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. **(S-MD.5.) (DOK 1,2,3)**
2. Find the expected payoff for a game of chance. For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.
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| 1. Evaluate and compare strategies on the basis of expected values. For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.
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| 1. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). **(S-MD.6.) (DOK 1,2)**
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| 1. Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game). **(S-MD.7.) (DOK 2,3)**
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| **Mathematics Depth-Of-Knowledge Definitions - Mathematics** |

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| *Level 1 (Recall of a fact or information procedure)* includes the recall of information such as a fact, definition, term, or a simple procedure, as well as performing a simple algorithm or applying a formula. That is, in mathematics a one-step, well-defined, and straight algorithmic procedure should be included at this lowest level. Other key words that signify a Level 1 include “identify,” “recall,” “recognize,” “use,” and “measure.” Verbs such as “describe” and “explain” could be classified at different levels depending on what is to be described and explained. Examples: |

* Recall or recognize a fact, term or property
* Represent in words, pictures or symbols in a math object or relationship
* Perform routine procedure like measuring

Level 2 (Basic Reasoning: Use information or conceptual knowledge, two or more steps) includes the engagement of some mental processing beyond a habitual response. A Level 2 assessment item requires students to make some decisions as to how to approach the problem or activity, whereas Level 1 requires students to demonstrate a rote response, perform a well-known algorithm, follow a set procedure (like a recipe), or perform a clearly defined series of steps. Keywords that generally distinguish a Level 2 item include “classify,” “organize,” ”estimate,” “make observations,” “collect and display data,” and “compare data.” These actions imply more than one step. For example, to compare data requires first identifying characteristics of the objects or phenomenon and then grouping or ordering the objects.

Some action verbs, such as “explain,” “describe,” or “interpret” could be classified at different levels depending on the object of the action. For example, if an item required students to explain how light affects mass by indicating there is a relationship between light and heat, this is considered a Level 2. Other Level 2 activities include explaining the purpose and use of experimental procedures; carrying out experimental procedures; making observations and collecting data; classifying, organizing, and comparing data; and organizing and displaying data in tables, graphs, and charts.

* Specify and explain relationships between facts, terms, properties or operations
* Select procedure according to criteria and perform it
* Solve routine multiple-step problems

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| *Level 3 (Complex Reasoning: Requires reasoning, developing a plan or a sequence of steps, working with some complexity, and considering more than one possible approach and answer)* requires reasoning, planning, using evidence, and a higher level of thinking than the previous two levels. In most instances, requiring students to explain their thinking is a Level 3. Activities that require students to make conjectures are also at this level. The cognitive demands at Level 3 are complex and abstract. The complexity does **not** result from the fact that there are multiple answers, a possibility for both Levels 1 and 2, but because the task requires more demanding reasoning. An activity, however, that has more than one possible answer and requires students to justify the response they give would most likely be a Level 3. Other Level 3 activities include drawing conclusions from observations; citing evidence and developing a logical argument for concepts; explaining phenomena in terms of concepts; and using concepts to solve problems. |

* Analyze similarities and differences between procedures
* Formulate original problem given situation
* Formulate mathematical model for complex situation

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| *Level 4 (Extended Reasoning: Requires an investigation, time to think and process multiple conditions of the problem)* requires complex reasoning, planning, developing, and thinking most likely over an extended period of time. The extended time period is **not** a distinguishing factor if the required work is only repetitive and does **not** require applying significant conceptual understanding and higher-order thinking. For example, if a student has to take the water temperature from a river each day for a month and then construct a graph, this would be classified as a Level 2. However, if the student is to conduct a river study that requires taking into consideration a number of variables, this would be a Level 4. At Level 4, the cognitive demands of the task should be high and the work should be very complex. Students should be required to make several connections—relate ideas within the content area or among content areas—and have to select one approach among many alternatives on how the situation should be solved, in order to be at this highest level. Level 4 activities include designing and conducting experiments; making connections between a finding and related concepts and phenomena; combining and synthesizing ideas into new concepts; and critiquing experimental designs.  |

* Apply mathematical model to illuminate a problem, situation
* Conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results
* Design a mathematical model to inform and solve a practical or abstract situation