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| **HS Mathematics: Functions – Planning Tool** |
| Collaborators:  | Academic Year: |
| *This planning tool can be used by collaborating teachers across a given school year or term to help insure full implementation of the Iowa Core Content Standards into their classroom instructional and assessment activities.* *Full implementation is accomplished when the district or school is able to provide evidence that an ongoing process is in place to ensure that each and every student is learning the standards and the essential concepts and skills of the Iowa Core. A school that has fully implemented the Iowa Core is engaged in an ongoing process of data gathering and analysis, decision making, identifying actions, and assessing the impact around alignment and professional development focused on content, instruction, and assessment. The school is fully engaged in a continuous improvement process that specifically targets improved student learning and performance.* ***Effective implementation of the Iowa Core is not a simple checklist. Implementation requires that educators strategically and systematically address the knowledge and skills being taught, engage in collaboration around the use of effective instructional practices and materials and develop activities to elicit evidence of student learning that match the level of rigor called for in the standards.*** |
| **Mathematic Content Standard** | **Aug.** | **Sept** | **Oct.** | **Nov.** | **Dec.** | **Jan.** | **Feb.** | **Mar** | **Apr.** | **May** |
| **Interpreting Functions: Understand the concept of a function and use function notation** |
| 1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). **(F-IF.1.) (DOK 1)**
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| 1. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. **(F-IF.2.) (DOK 1,2)**
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| 1. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1. **(F-IF.3.) (DOK 1)**
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| **Interpreting Functions: Interpret functions that arise in applications in terms of the context**  |
| 1. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.★ **(F-IF. 4.) (DOK 1,2)**
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| 1. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.★ **(F-IF.5.) (DOK 1,2)**
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| 1. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. **(F-IF.6.)**
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| **Interpreting Functions: Analyze functions using different representations** |
| 1. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. **(F-IF.7.) (DOK 1,2)**
	1. Graph linear and quadratic functions and show intercepts, maxima, and minima.

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| 1. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
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| 1. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
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| 1. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
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| 1. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude
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| 1. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. **(F-IF.8.) (DOK 1,2)**
	1. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
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| 1. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay.
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| 1. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. **(F-IF.9.) (DOK 1,2)**
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| **Building Functions: Build a function that models a relationship between two quantities** |
| 1. Write a function that describes a relationship between two quantities.★ **(F-BF.1.) (DOK 1,2)**
	1. Determine an explicit expression, a recursive process, or steps for calculation from a cntext.
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| 1. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.
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| 1. (+) Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.
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| 1. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★ **(F-BF.2.) (DOK 1,2)**
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| **Building Functions: Build new functions from existing functions** |
| 1. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. **(F-BF.3) (DOK 1,2)**
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| 1. Find inverse functions. **(F-BF.4) (DOK 1,2)**
	1. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) =2 x3 or f(x) = (x+1)/(x–1) for x ≠ 1.
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| 1. (+) Verify by composition that one function is the inverse of another.
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| 1. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
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| 1. (+) Produce an invertible function from a non-invertible function by restricting the domain.
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| 1. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. **(F-BF.5) (DOK 1,2)**
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| **Linear, Quadratic, and Exponential Models: Construct and compare linear, quadratic, and exponential models and solve problems**  |
| 1. Distinguish between situations that can be modeled with linear functions and with exponential functions. **(F-LE.1.) (DOK 1,2,3)**

 Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.  |  |  |  |  |  |  |  |  |  |  |
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| 1. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
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| 1. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
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| 1. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). **(F-LE.2.) (DOK 1,2)**
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| 1. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. **(F-LE.3.) (DOK 1,2)**
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| 1. For exponential models, express as a logarithm the solution to abct = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. **(F-LE.4.) (DOK 1)**
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| **Linear, Quadratic, and Exponential Models: Interpret expressions for functions in terms of the situation they model** |
| 1. Interpret the parameters in a linear or exponential function in terms of a context. **(F-LE.5.) (DOK 1,2)**
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| **Trigonometric Functions: Extend the domain of trigonometric functions using the unit circle**  |
| 1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. **(F-TF.1.) (DOK 1)**
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| 1. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. **(F-TF.2.) (DOK 1,2)**
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| 1. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for π–x, π+x, and 2π–x in terms of their values for x, where x is any real number. **(F-TF.3.) (DOK 1,2)**
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| 1. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. **(F-TF.4.) (DOK 2)**
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| **Trigonometric Functions: Model periodic phenomena with trigonometric functions** |
| 1. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★ **(F-TF.5.) (DOK 1,2)**
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| 1. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. **(F-TF.6.) (DOK 1,2)**
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| 1. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context**. (F-TF.7.) (DOK 1,2,3)**
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| **Trigonometric Functions: Prove and apply trigonometric identities** |
| 1. Prove the Pythagorean identity sin2(θ) + cos2(θ) = 1 and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle. **(F-TF.8.) (DOK 1,2,3)**
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| 1. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. **(F-TF.9.) (DOK 1,2,3)**
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| **Mathematics Depth-Of-Knowledge Definitions - Mathematics** |

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| *Level 1 (Recall of a fact or information procedure)* includes the recall of information such as a fact, definition, term, or a simple procedure, as well as performing a simple algorithm or applying a formula. That is, in mathematics a one-step, well-defined, and straight algorithmic procedure should be included at this lowest level. Other key words that signify a Level 1 include “identify,” “recall,” “recognize,” “use,” and “measure.” Verbs such as “describe” and “explain” could be classified at different levels depending on what is to be described and explained. Examples: |

* Recall or recognize a fact, term or property
* Represent in words, pictures or symbols in a math object or relationship
* Perform routine procedure like measuring

Level 2 (Basic Reasoning: Use information or conceptual knowledge, two or more steps) includes the engagement of some mental processing beyond a habitual response. A Level 2 assessment item requires students to make some decisions as to how to approach the problem or activity, whereas Level 1 requires students to demonstrate a rote response, perform a well-known algorithm, follow a set procedure (like a recipe), or perform a clearly defined series of steps. Keywords that generally distinguish a Level 2 item include “classify,” “organize,” ”estimate,” “make observations,” “collect and display data,” and “compare data.” These actions imply more than one step. For example, to compare data requires first identifying characteristics of the objects or phenomenon and then grouping or ordering the objects.

Some action verbs, such as “explain,” “describe,” or “interpret” could be classified at different levels depending on the object of the action. For example, if an item required students to explain how light affects mass by indicating there is a relationship between light and heat, this is considered a Level 2. Other Level 2 activities include explaining the purpose and use of experimental procedures; carrying out experimental procedures; making observations and collecting data; classifying, organizing, and comparing data; and organizing and displaying data in tables, graphs, and charts.

* Specify and explain relationships between facts, terms, properties or operations
* Select procedure according to criteria and perform it
* Solve routine multiple-step problems

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| *Level 3 (Complex Reasoning: Requires reasoning, developing a plan or a sequence of steps, working with some complexity, and considering more than one possible approach and answer)* requires reasoning, planning, using evidence, and a higher level of thinking than the previous two levels. In most instances, requiring students to explain their thinking is a Level 3. Activities that require students to make conjectures are also at this level. The cognitive demands at Level 3 are complex and abstract. The complexity does **not** result from the fact that there are multiple answers, a possibility for both Levels 1 and 2, but because the task requires more demanding reasoning. An activity, however, that has more than one possible answer and requires students to justify the response they give would most likely be a Level 3. Other Level 3 activities include drawing conclusions from observations; citing evidence and developing a logical argument for concepts; explaining phenomena in terms of concepts; and using concepts to solve problems. |

* Analyze similarities and differences between procedures
* Formulate original problem given situation
* Formulate mathematical model for complex situation

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| *Level 4 (Extended Reasoning: Requires an investigation, time to think and process multiple conditions of the problem)* requires complex reasoning, planning, developing, and thinking most likely over an extended period of time. The extended time period is **not** a distinguishing factor if the required work is only repetitive and does **not** require applying significant conceptual understanding and higher-order thinking. For example, if a student has to take the water temperature from a river each day for a month and then construct a graph, this would be classified as a Level 2. However, if the student is to conduct a river study that requires taking into consideration a number of variables, this would be a Level 4. At Level 4, the cognitive demands of the task should be high and the work should be very complex. Students should be required to make several connections—relate ideas within the content area or among content areas—and have to select one approach among many alternatives on how the situation should be solved, in order to be at this highest level. Level 4 activities include designing and conducting experiments; making connections between a finding and related concepts and phenomena; combining and synthesizing ideas into new concepts; and critiquing experimental designs.  |

* Apply mathematical model to illuminate a problem, situation
* Conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results
* Design a mathematical model to inform and solve a practical or abstract situation